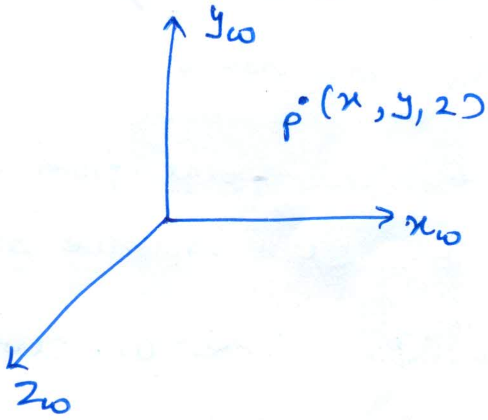
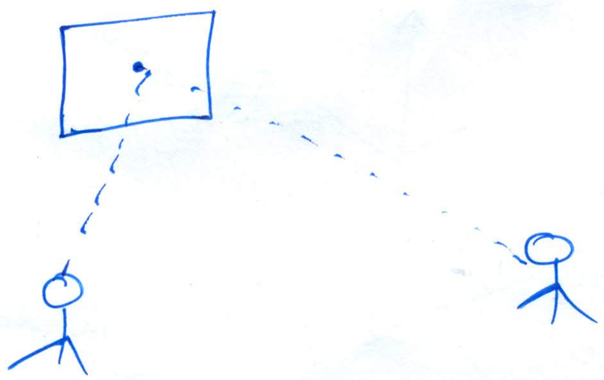
Elementary 3-D graphics →



① An object in 3D can be viewed from different angle or viewing position such as front, back, top. or side view.

② To view to object in 2D It is firstly mapped in viewing coordinate from world co-ordinate.

③ In order to choose a particular view for scene, we first establish the viewing co-ordinate. It is also known as view reference co-ordinate system.

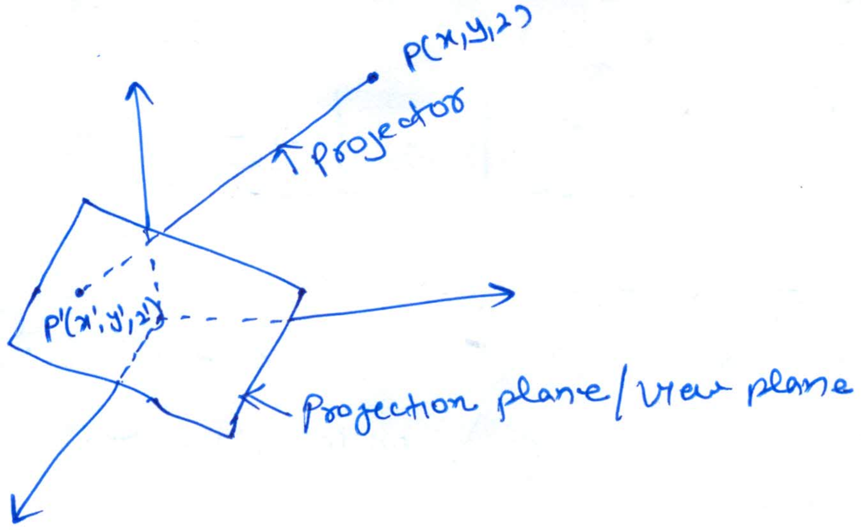


Projection → It is process of representing 3-D object into 2-D object.

Basic terms related to projection →

1. Centre of Projection → The point from where projection is taken. It can either be light source or eye position
2. Projection plane → It is plane or surface onto which the projection of object is formed. or view plane
3. Projector → Lines emerging from centre of projection are called projectors.

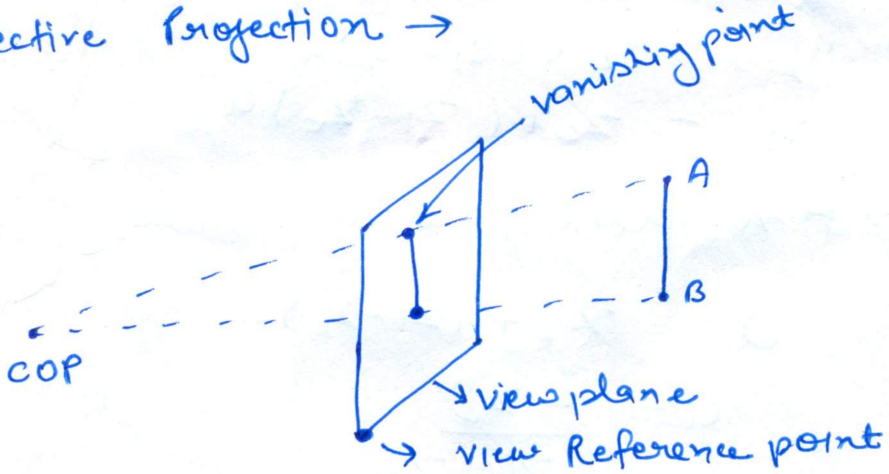
Thus projection can be defined as: mapping of point $P(x, y, z)$ onto its image $P'(x', y', z')$ in projection plane or view plane. This mapping is determined by projection line called projector that passes through P and intersects the view plane at P'



Types of Projection

- (1) Perspective Projection
  - (a) → One point
  - (b) → Two point
  - (c) → Three point
- (2) Parallel Projection
  - (2.1) orthographic projection
    - (a) Multiview
    - (b) Axonometric
      - → Isometric
      - → Trimetric
      - → diametric
  - (2.2) oblique Projection
    - (a) Cavalier
    - (b) Cabinet

1. Perspective Projection →



① vanishing point → The point at which a set of projected parallel lines appears to converage is called vanishing point.

② It produces realistic views but does not preserve the ~~real~~ relative proportions of an object dimension

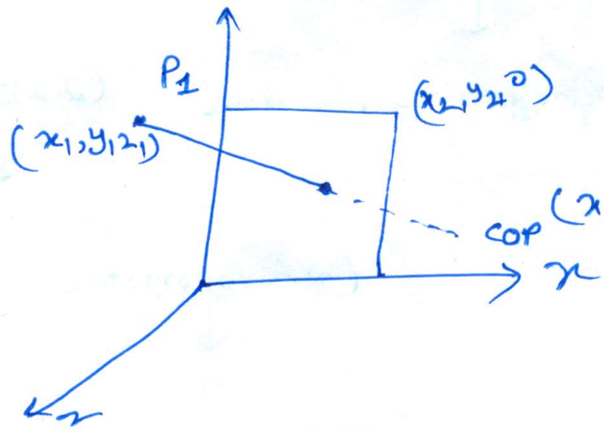
③ Projection of distant objects are smaller than projection of objects of same size that are closer to the projection plane

Transformation Matrix for Perspective Projection

COP — x_c, y_c, z_c

Point on object (P_1) — (x_1, y_1, z_1)

Point on projection plane — (x_2, y_2, z_2)



Parametric eqn for line containing these points

$$x_2 = x_c + (x_1 - x_c)u$$
$$y_2 = y_c + (y_1 - y_c)u$$
$$z_2 = z_c + (z_1 - z_c)u$$

on projection plane, $z = 0$

$$0 = z_c + (z_1 - z_c)u$$

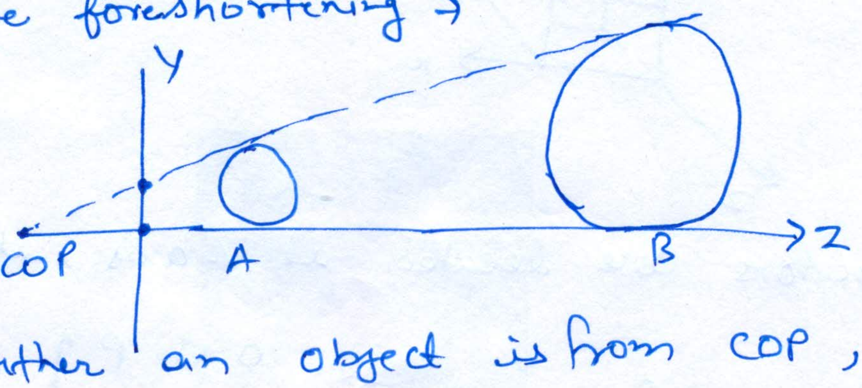
$$u = \frac{-z_c}{z_1 - z_c}$$

$$x_2 = \frac{x_c z_1 - x_1 z_c}{(z_1 - z_c)} \qquad y_2 = \frac{y_c z_1 - y_1 z_c}{(z_1 - z_c)}$$

$$[x_2 \ y_2 \ z_2 \ 1] = [x_1 \ y_1 \ z_1 \ 1] \begin{bmatrix} -z_c & 0 & 0 & 0 \\ 0 & -z_c & 0 & 0 \\ x_c & y_c & 0 & 1 \\ 0 & 0 & 0 & -z_c \end{bmatrix}$$

Perspective Anomalies → which distorts actual size and shape.

1. perspective foreshortening →



The farther an object is from COP, the smaller it appears ie its projected size becomes smaller.

2. Vanishing point → Railroad track appear to meet at a point in horizon

Types of Perspective Projection →

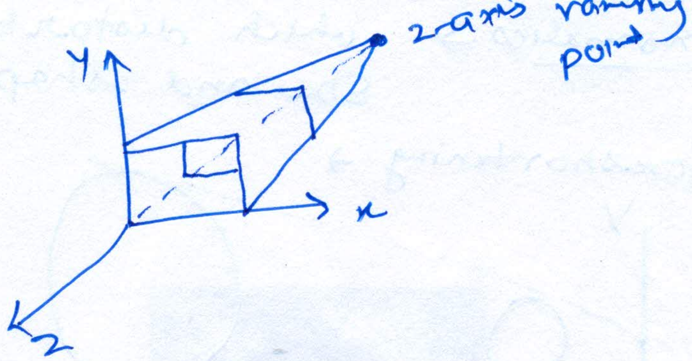
These are classified on basis of no. of vanishing points. and vanishing point is a pt in ~~which~~ image where parallel line through the COP intersects the view plane.

① One point Perspective Projection →

→ It occurs when any one principal axes. intersects with projection plane.

→ In other word ~~one~~ when the projection plane is perpendicular to one of its principal axes.

for eg → only a z-axis vanishing point occurs when the z-axis intersects the projection plane whereas x and y and xy plane remains parallel to the projection plane.



when projectors are located at x-axis, it is given by

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \; y \; z \; (px+1)]$$

$$[x' \; y' \; z' \; 1] = (px+1) \left[\frac{x}{(px+1)} \; \frac{y}{px+1} \; \frac{z}{(px+1)} \; 1 \right]$$

$$px+1 \neq 1$$

$$COP = [-\tfrac{1}{p} \; 0 \; 0 \; 1]$$

$$VP = [\tfrac{1}{p} \; 0 \; 0 \; 1]$$

for y axis $[x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

for z-axis $[x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Two point Projection →

→ It occurs when projection plane intersects exactly two of the principal axes.

For eg when projection plane intersects the x & y axes and the z axes remains parallel to projection plane, two vanishing points are formed.

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \quad y \quad z \quad px + qy + 1]$$

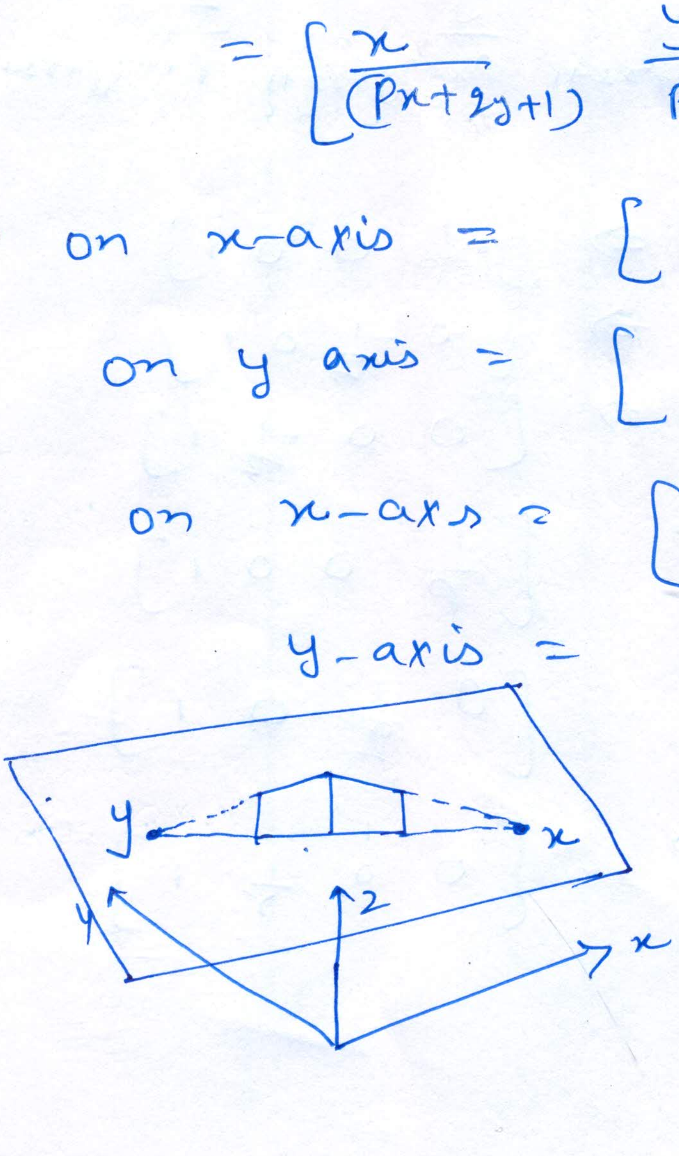
$$= \left[\frac{x}{(px+qy+1)} \quad \frac{y}{px+qy+1} \quad \frac{z}{px+qy+1} \quad 1 \right]$$

COP on x-axis = $[-\frac{1}{p} \quad 0 \quad 0 \; 1]$

COP on y axis = $[0 \quad -\frac{1}{q} \quad 0 \quad 1]$

VOP on x-axis = $[\frac{1}{p} \quad 0 \quad 0 \; 1]$

y-axis = $[0 \quad \frac{1}{q} \quad 0 \; 1]$



Three point perspective Projection

- It occurs when the projection plane intersects all three of the principal axes x, y, z
- none of principal axes is parallel to the projection plane.

for eg

$$[x'\ y'\ z'\ 1] = [x\ y\ z\ 1]\begin{bmatrix}1 & 0 & 0 & p\\ 0 & 1 & 0 & q\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$= [x\quad y\quad z\quad px+qy+rz+1]$$

$$= \left[\frac{x}{px+qy+rz+1}\quad \frac{y}{px+qy+rz+1}\quad \frac{z}{px+qy+rz+1}\quad 1\right]$$

COP on x-axis $\Rightarrow \left[-\frac{1}{p}\ 0\ 0\ 1\right]$

y-axis $\Rightarrow \left[0\ -\frac{1}{q}\ 0\ 1\right]$

z-axis $\Rightarrow \left[0\ 0\ -\frac{1}{r}\ 1\right]$

VP on x-axis $\left[\frac{1}{p}\ 0\ 0\ 1\right]$

y-axis $\left[0\ \frac{1}{q}\ 0\ 1\right]$

z-axis $\left[0\ 0\ \frac{1}{r}\ 1\right]$

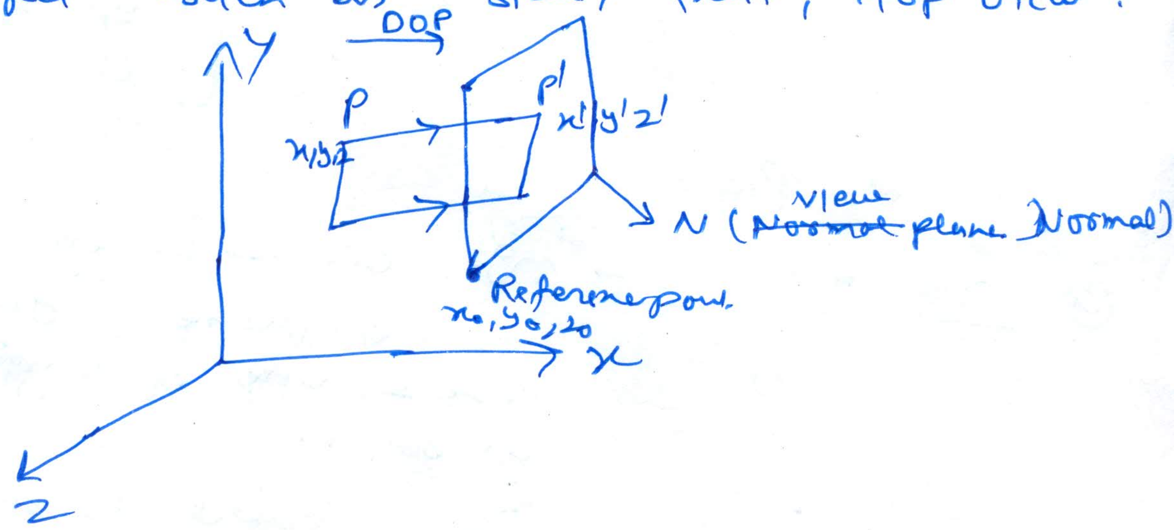
Parallel Projection

→ COP is at infinite distance from Projection plane. & projector are parallel to each other

→ The direction of projection is prescribed direction for all projectors.

→ It preserves the relative proportions of an object.

→ It is used to produce various views of an object such as side, front, top view.



Transformation Matrix for Parallel Projection,

→ DOP is given by vector (x_p, y_p, z_p)

→ Let point on object is $P_1(x_1, y_1, z_1)$

Now to determine the location of projected point on projection plane, we consider parametric eqn of line passing through P_1 and the DOP such as

$$x_2 = x_1 + x_p u$$
$$y_2 = y_1 + y_p u$$
$$z_2 = z_1 + z_p u$$

On projection plane $z=0$

then $u = \frac{-z_1}{z_p}$

Now by substituting the value of u we get

$$x_2 = x_1 - z_1\left[\frac{x_p}{z_p}\right]$$

$$y_2 = y_1 - z_1\left[\frac{y_p}{z_p}\right]$$

homogeneous Matrix →

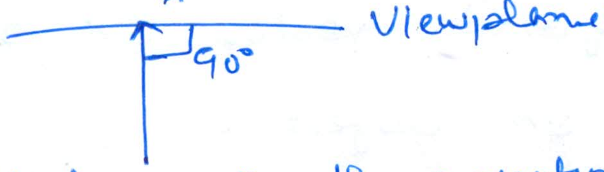
$$[x_2\ y_2\ z_2\ 1] = [x\ y\ z\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_p}{y_p} & \frac{y_p}{x_p} & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Types of Projection →

(a) orthographics → (1) It occurs when DOP is ⊥ to view plane. It produce true size & shape of single plane face of an object.

(2) Projection plane is parallel to any one of planes xy plane, yz plane, xz plane.

For eg



orthographics projection is the projection on one of the co-ordinate planes i.e $x=0, y=0, z=0$.

The Matrix for Projection onto the $x=0$, yz plane

$$P_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

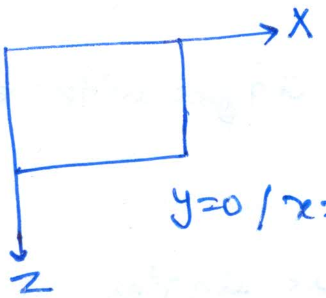
xz plane xy plane

Orthographis Projection →

(a) Multiview Orthographics

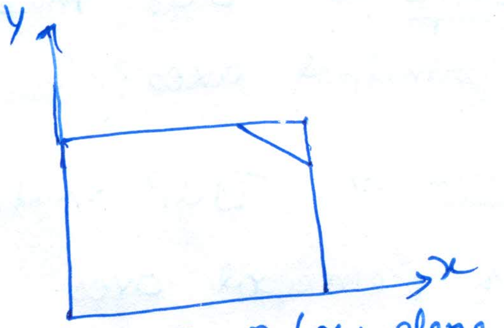
(b) Axonometric Orthographic

→ When the DOP is parallel to any of the principal axes, it produces the front, top, rear and views of the object.



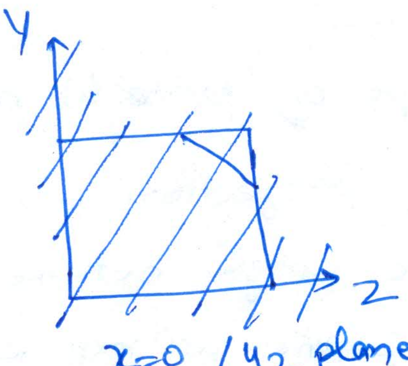
y=0 / xz plan

Plan/top view



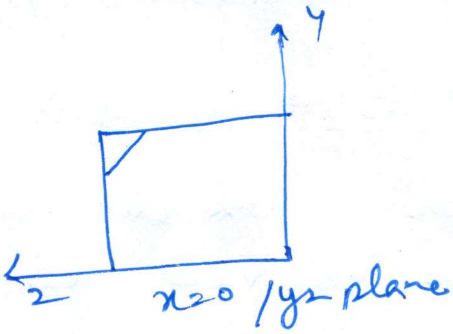
z=0 / xy plane

Rear View



x=0 / yz plane

side View



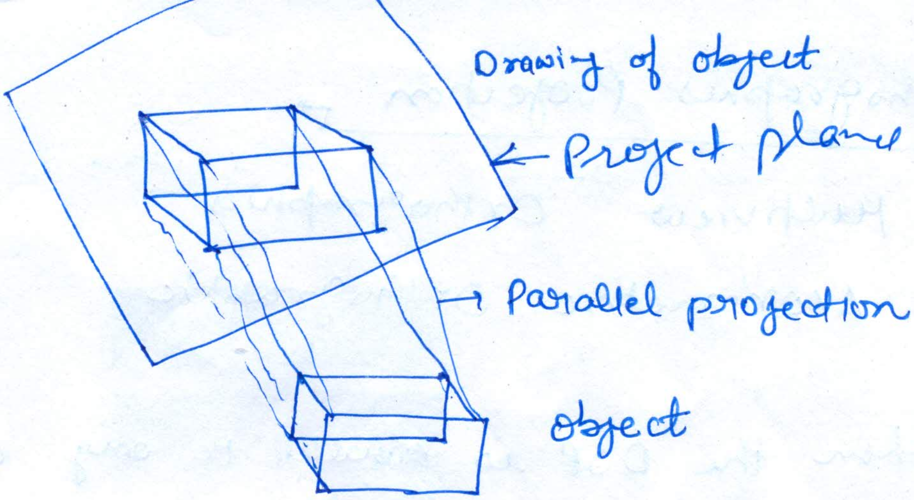
x=0 / yz plane

Axonometric Orthographic Projection →

→ When DOP is not parallel to any of three principal axes rather than DOP is perpendicular to view plane

→ It is used to overcome the limitation of single orthographics projection which is not able to show the general 3-D shape of an object.

for eg



① **Isometric** → DOP makes equal angles with all the three principal axes.

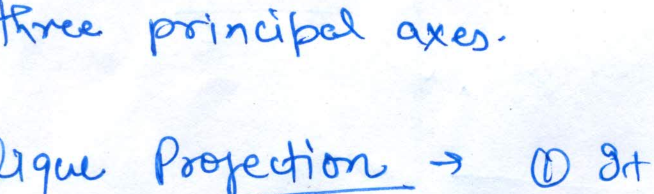
② **Diametric** → DOP makes equal angles with exacly two of principal axes.

③ **Trimetric** → DOP makes unequal angles with the three principal axes.

Oblique Projection → ① It is a type of parallel projection

② It occurs when direction of projection is not perpendicular to view plane i.e angle between the projectors and the projection plane is not equal to 90°.

③ It combines the features of both multiview and axonometric llel projection.



1. Cavalier projection
2. Cabinet projection

Case study on Perspective & parallel projection